

Interaction of an electromagnetic wave with a rapidly created spatially periodic plasma

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The interaction of electromagnetic waves with rapidly created time-varying spatially periodic plasmas is studied. The numerical results of the collisionless case show that both frequency upshifted and frequency downshifted waves are generated. Moreover, the frequency downshifted waves are trapped by the plasma when the plasma frequency is larger than the wave frequency. The trapping has the effect of dramatically enhancing the efficiency of the frequency downshift conversion process, by accumulating incident wave energy during the plasma transition period. A theory based on the wave impedance of each Floquet mode of the periodic structure is formulated, incorporating with the collisional damping of the plasma. Such a theory explains the recent experimental observations [Faith, Kuo, and Huang, *Phys. Rev. E* **55**, 1843 (1997)] where the frequency downshifted signals were detected repetitively with considerably enhanced spectral intensities while the frequency upshifted signals were missing. [S1063-651X(97)13408-0]

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I. INTRODUCTION

The propagation of electromagnetic waves in linear dielectric media has always been the subject of interest. Due to the high speed of the electromagnetic wave, it is more likely for the wave to encounter spatial variation of the medium during its propagation than to experience the medium's temporal variation. Therefore, in the normal situation, the spatial domain wave phenomena, such as reflection, refraction, and transmission, etc. resulting from the change of the wave vector, dominate over the time domain wave phenomena. For such a case the wave frequency can be assumed to be unchanged and the analysis can be simplified by considering time harmonic problems [1]. However, for the case when the medium varies rapidly in time over a sizable dimension (reference to the wavelength), the time domain phenomena resulting from frequency change may emerge. From space-time duality, it seems that some analogies between phenomena in two domains may be drawn straightforwardly [2,3]. It is indeed so for many of them, but some fundamental differences also exist. For example, the causality concept forbids the time domain reflection phenomenon, the sharp discontinuity in the spatial domain does not exist in reality in the time domain, and most significantly, a wave can preexist in a time-varying medium but it has to propagate into a spatially varying medium.

The theory, assuming instantaneous plasma generation in an infinite medium, has been worked out by Jiang [4] and Wilks *et al.* [5]. They find that the wave number stays fixed, the frequency is upshifted, and the initially forward propagating wave splits up into a forward and a backward propagating component at the higher frequency. A phenomenon that is not foreseen by the duality is the generation of a static magnetic field oscillating in space, with the same wave number. More recently theory has been extended to consider a plasma slab [6,7]. As the transition period of the medium

(e.g., finite creation time of plasma) increases, it is shown numerically that the frequency spectra of new waves are broadened [5,6], a phenomenon different from that in the spatial domain where the wave number spectra of new waves outside the transition region are not broadened. Frequency upshifting and spectral broadening of a wave interacting with a rapidly created plasma slab have also been demonstrated experimentally [8–13]. An additional phenomenon observed in the experiments is the unexpected frequency downshift together with frequency upshift leading to spectral breaking of the new pulse [14]. It is then verified that the energy loss of the wave caused by the new medium during the finite transition period is responsible for the frequency downshifting result [14].

In a recent experiment [15] a spatially periodic plasma is generated by a set of discharges, where the plasma frequency is time dependent. The plasma in the discharge first increases and then decreases, with the time scales not necessarily short compared with the wave frequency scale. Normally, one would expect that the periodic structure would introduce Floquet modes [16] into the system so that one can upshift waves to frequencies greater than a single slab of the same plasma density as well as downshift waves to frequencies in the cutoff region of a single plasma slab. The frequency downshifted waves have indeed been observed in the experiment, however, the frequency upshifted waves are missing [15] (i.e., they cannot be distinguished conclusively from the noise). Moreover, the frequency downshifted waves were found to be trapped in the periodic structure so that their spectral intensities were accumulating during the transition period and hence, enhanced significantly [15] (20–50 dB above the noise level). In this work, a theory explaining the trapping phenomenon is presented. The theory will also explain why frequency upshifted waves are missing in the experimental measurements.

II. THEORY AND FORMULATION

Considering a one-dimensional wave propagation in an unmagnetized plasma varying arbitrarily in both space and

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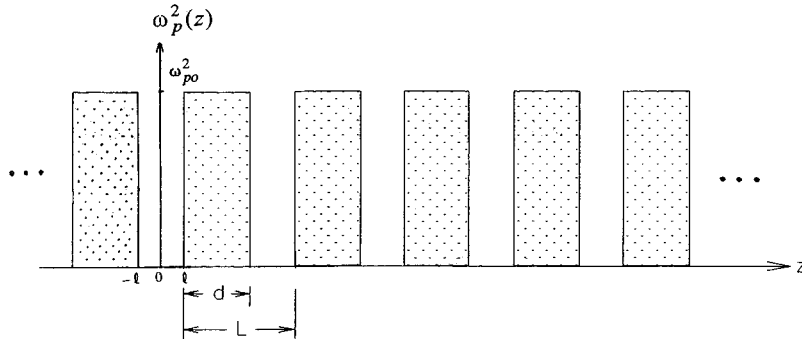


FIG. 1. Geometry of the periodic plasma structure (after plasma creation) represented by the spatial distribution of the plasma frequency $\omega_p(z)$.

time, the wave equation is given by

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\omega_p^2(z, t)}{c^2} \right] \epsilon(z, t) = 0. \quad (1)$$

In the present work, we are interested in the case in which the plasma is created rapidly at $t=0$ and then decays slowly. It is also structured in the form of parallel slabs. The plasma frequency is, thus, modeled as

$$\begin{aligned} \omega_p^2(z, t) &= 0 \quad \text{for } t < 0 \\ \omega_p^2(z, t) &= \omega_{p0}^2 [1 - \exp(-t/\tau_r)] \exp(-t/\tau_f) \\ &\quad \times \sum_{n=0}^{N-1} P_{d/2}(z - nL - d/2) \quad \text{for } t > 0, \end{aligned} \quad (2)$$

where τ_r and τ_f are the plasma rise and decay times, respectively, and $\tau_f \gg \tau_r$; $P_{d/2}(z - a)$ is a unit rectangular pulse of width d centered about $z = a$, L is the separation between two adjacent slabs and $L \geq d$, and N is the total number of plasma slabs in the structure. A case considering a sinusoidal density profile $\omega_p^2 = \omega_{p2}^2 \cos^2 \pi z/L$ changing its magnitude

ω_{p2}^2 from ω_{p0}^2 to ω_{p1}^2 at $t=0$ abruptly has been studied by Schmidt *et al.* [17]. The wave equation is in the form of Mathieu's equation.

As N is large enough, Eq. (2) resembles a periodic structure having a spatial period L . Thus, one may adopt the electromagnetic property of a periodic dielectric medium for such a structured plasma as a qualitative base. It is manifested by the dispersion relations of the Floquet modes propagating in a periodic dielectric medium, which form a band diagram similar to that for electron waves in solids [18]. This diagram allows one to predict the frequencies of all the Floquet modes at a given wave number. They are expected to correspond to the frequencies of the peaks in the power spectrum of the resultant wave after interacting with a rapidly created periodic plasma described by Eq. (2). We simplify the situation to an infinite periodic structure with an abrupt creation of the plasma at $t=0$ and no plasma decay afterward by letting n run from $-\infty$ to ∞ , $\tau_r \rightarrow 0^+$, and $\tau_f \rightarrow \infty$ in Eq. (2), and assume a time harmonic wave (with $\exp[-(i\omega t)]$ time dependence). The phasor solution of Eq. (1) for $t > 0$ in one spatial period $-l < z < l + d$ as shown in Fig. 1, where $l = (L - d)/2$, can be written as

$$E(z) = \begin{cases} A \exp(ikz) + B \exp(-ikz), & -l < z < l \\ C \exp[ik\eta(z-l)] + D \exp[-ik\eta(z-l)], & l < z < l + d, \end{cases} \quad (3)$$

where $k = \omega/c$ and $\eta = (1 - \omega_{p0}^2/\omega^2)^{1/2}$ are the free space wave number and the index of refraction of the plasma slab, respectively. Applying the boundary conditions at $z = \pm l$, $E(\pm l_-) = E(\pm l_+)$ and $\partial_z E(\pm l_-) = \partial_z E(\pm l_+)$ and using the Bloch wave condition [18]

$$E(z) = \exp(-i\beta L) E(z + L) \quad (4)$$

to replace $E(-l_-)$ by $e^{-i\beta L} E[(l+d)_-]$, where $E[(l+d)_-]$ is given by Eq. (3) and β is the propagation constant for the periodic structure as a whole, four algebraic equations are obtained. Three of them can be solved to express B , C , and D in terms of A as follows:

$$B = -(b - \sqrt{b^2 - 1})A,$$

$$C = \frac{1}{2\eta} [(\eta + 1)e^{ikl} - (\eta - 1)(b - \sqrt{b^2 - 1})e^{-ikl}]A,$$

$$D = \frac{1}{2\eta} [(\eta - 1)e^{ikl} - (\eta + 1)(b - \sqrt{b^2 - 1})e^{-ikl}]A, \quad (5)$$

where

$$b = [1/(\eta^2 - 1)][(1 + \eta^2)\cos 2kl + 2\eta \cot k\eta d \sin 2kl].$$

This leads the fourth equation to the dispersion relation

$$\cos \beta L = \cos k\eta d \cos 2kl - \frac{1}{2}(\eta + 1/\eta) \sin k\eta d \sin 2kl. \quad (6)$$

Since $\cos \beta L$ is an even and periodic function, Eq. (6) has to be solved only for $0 \leq \beta L \leq \pi$. Equation (6) has multiple roots, $\omega_j(\beta)$, $j = 1, 2, \dots$; each root contributes to a branch

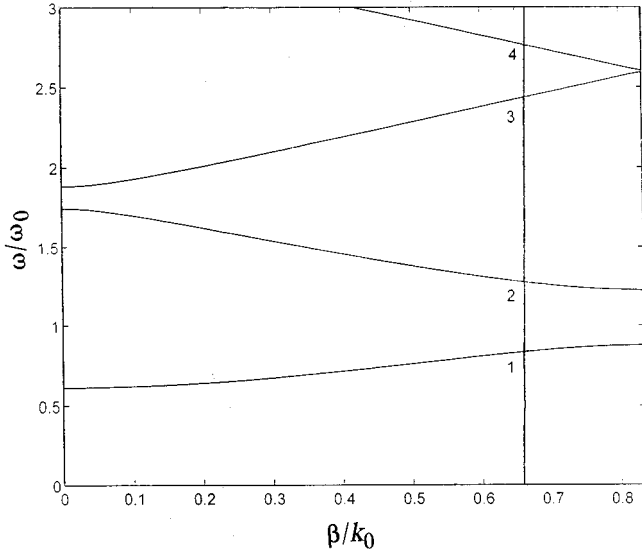


FIG. 2. Dispersion relation $\omega(\beta)$ for the case $L = -0.6\lambda_0$, and $\omega_{p0} = 1.2\omega_0$, where λ_0 and $f_0 = \omega_0/2\pi$ are the wavelength and frequency of a reference wave (initial wave) in free space. The vertical line is at $\beta/k_0 = 2\pi/k_0 L - 1 = 0.667$ and its intersecting points with the dispersion curves determine the frequencies of the Floquet modes converted from the initial reference wave (ω_0, k_0) after interacting with the suddenly created periodic plasma.

of the dispersion curves and covers a frequency range as a pass band. Shown in Fig. 2 is an example of the dispersion curves for the case $L = 0.6\lambda_0$, $d = 0.2\lambda_0$, and $\omega_{p0} = 1.2\omega_0$, where λ_0 and $f_0 = \omega_0/2\pi$ are the wavelength and frequency of a reference wave in free space. The frequency gap between two adjacent branches of the dispersion curves forms a stop band, or band gap, which is one of the characteristic features of periodic structures. Hence, for a given βL , there exists an infinite number of Floquet modes oscillating at discrete frequencies $\omega_j(\beta)$. Substituting $k = \omega_j(\beta)/c$ into Eqs. (3) and (5), and using Eq. (5) to replace the coefficients B , C , and D in Eq. (3) in terms of A , the field distribution of the j th Floquet mode is, thus, obtained.

We now consider a case where a wave exists everywhere in free space prior to the plasma creation, i.e.,

$$\epsilon(z, t \leq 0) = E_0 \cos(k_0 z - \omega_0 t), \quad (7)$$

where $k_0 = \omega_0/c$, and a periodically structured plasma characterized by Eq. (2) with $\tau_r \rightarrow 0$, $\tau_f \rightarrow \infty$, and n running from $-\infty$ to ∞ , is suddenly created at $t = 0$. Since the plasma is created instantaneously, the spatial distribution of the wave field $\epsilon(z, t = 0^+) = E_0 \cos k_0 z$ will not be changed during that instant. In order to propagate in the new medium at $t > 0$, the wave has to satisfy the new dispersion relation (6). Thus, an infinite number of Floquet modes oscillating at discrete frequencies $\omega_j(\beta)$, where $0 \leq \beta = k_0 - 2m\pi/L \leq \pi/2$ and m is an integer, are generated, i.e.,

$$\epsilon(z, t > 0) = \sum_{j=1}^{\infty} A_{j\pm} \cos(k_0 z \mp \omega_j t), \quad (8)$$

where $\sum_{j=1}^{\infty} A_{j\pm} = E_0$ is given by the initial condition. From Faraday's law $\vec{\nabla} \times \vec{E} = -(1/c)(\partial/\partial t)\vec{B}$, the corresponding wave magnetic field \vec{B}_1 is obtained to be

$$B_1(z, t > 0) = \sum_{j=1}^{\infty} \pm (k_0 c / \omega_j) A_{j\pm} \cos(k_0 z \mp \omega_j t). \quad (9)$$

However, Eq. (9) cannot satisfy the initial condition $B_1(z, t = 0) = (k_0 c / \omega_0) E_0 \cos k_0 z$. Therefore, a wiggler magnetic field $B_w(z) = [(k_0 c / \omega_0) E_0 - \sum_{j=1}^{\infty} \pm (k_0 c / \omega_j) A_{j\pm}] \cos k_0 z$, in addition to Eq. (9), is also excited in the new medium.

In the following we will show, via numerical analysis of Eq. (1), that the frequencies of the peaks of the power spectrum of the wave (7) after interacting with a rapidly created periodic plasma (2) do indeed correspond to the frequencies of the Floquet modes determined by Eq. (6). However, each spectral peak is broadened by the finite size of the plasma structure and the finite temporal variation of the plasma density. The numerical analysis will also reveal a wave trapping process. The trapped incident wave accumulates its energy during the finite growth period of the plasma and then emerges during the decaying period of the plasma. It leads to a significant enhancement of the spectral intensity of the frequency downshifted lines, a phenomenon that is not obvious from the band diagram but can be explained in terms of wave impedance described later.

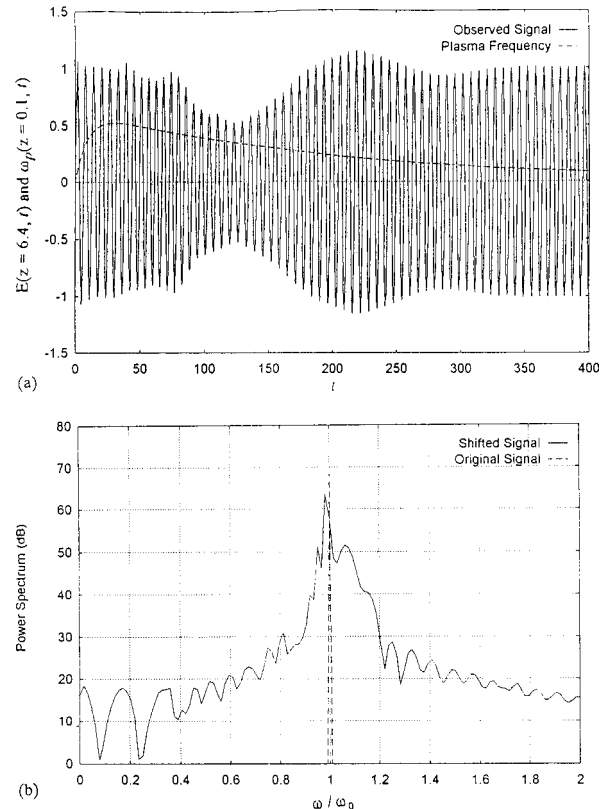


FIG. 3. (a) Time dependence of the observed signal outside the plasma structure with $N = 11$, $L = 0.6$, $d = 0.2$, $\omega_{p0} = 0.8$, $\alpha = 0.1$, and $\gamma = 0.002$, and (b) its power spectrum.

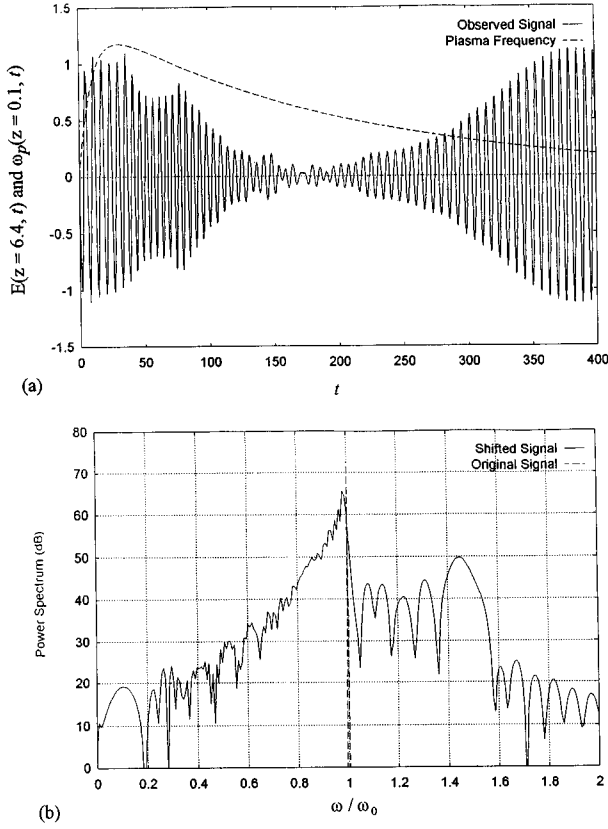


FIG. 4. (a) Time dependence of the observed signal outside the plasma structure with $N=11$, $L=0.6$, $d=0.2$, $\omega_{p0}=1.2$, $\alpha=0.1$, and $\gamma=0.002$, and (b) its power spectrum.

Frequency shifting and wave trapping

Considering the density profile (2), Eq. (1) is solved numerically to demonstrate frequency shifting and wave trapping phenomena. To carry out numerical analysis, it is helpful to first normalize all the relevant parameters to dimensionless forms. Convenient definitions of the normalized variables are

$$d/\lambda_0 \rightarrow d, \quad L/\lambda_0 \rightarrow L, \quad \omega_0 t/2\pi \rightarrow t, \quad \omega_{p0}/\omega_0 \rightarrow \omega_{p0},$$

$$\alpha = 2\pi/\omega_0 \tau_f, \quad \gamma = 2\pi/\omega_0 \tau_f.$$

For the remainder of this work we will refer to the dimensionless parameters unless units are explicitly given. A finite difference time domain method is needed to solve the wave equation (1) for computing the observed field (in time) at a particular spatial location. This time series is subsequently fast Fourier transformed (FFT) and the power spectrum found by multiplying the time series' FFT by its complex conjugate.

We now consider a specific example to explain the experimental observations [15] on frequency shifting and wave trapping phenomena. Thus, $\omega_0 = 2\pi \times 3.1$ GHz, $N=11$, $\alpha=0.1$, $\gamma=2 \times 10^{-3}$, $L=0.6$, and $d=0.2$. Two cases of $\omega_{p0}=0.8$ and 1.2 , respectively, will be analyzed for comparison and discussion. Figure 2 is the dispersion relation of the ideal structure (i.e., $N \rightarrow \infty$, $\alpha \rightarrow \infty$, and $\gamma=0$) for $\omega_{p0}=1.2$. In Figs. 3–7 we present several representations of the shifted signals observed in time and their power spectra computed

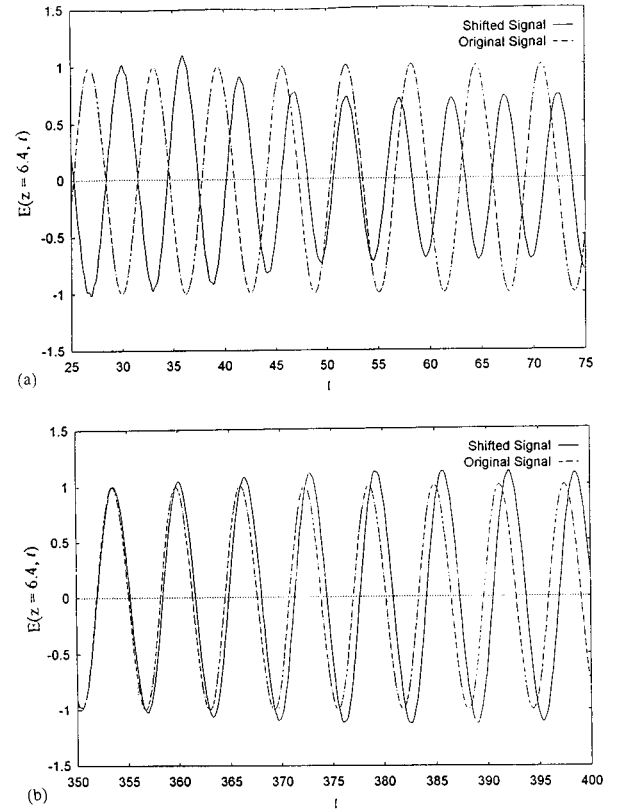


FIG. 5. Observed and incident signals of the previous figure, (a) at early time showing upshifted waves, and (b) at later time showing downshifted waves.

under different conditions. All the time plots display the amplitude of the observed signals, normalized to the incident signal amplitude. The spectral plots show the power spectra of the observed time series of the signals as measured inside or outside the plasmas and of the incident wave as a reference. In the case of Fig. 3, the observation point is chosen outside the plasma, and $\omega_{p0}=0.8$. We see that the observed signal has only a small dip in amplitude, resulting from most of the incident wave being able to propagate through the plasma, with only a small fraction of wave power trapped. The power spectrum shows a prominent upshifted lobe due to the growth, and a smaller downshifted lobe due to the later decay. In Fig. 4 the only change from the previous case is that now $\omega_{p0}=1.2$. The time plot shows that initially the observed signal is of large amplitude, decays to a small level, and later grows again. This can be explained as follows. Immediately after the plasma is created the upshifted waves propagate out of the plasma. This explains the first part of the waveform with large amplitude. Once these waves are clear of the plasma the signal amplitude drops as the downshifted waves remain trapped. However, as the plasma decays, the incident wave can propagate through again and more and more of the downshifted waves can escape, and both contribute to the later rise in signal amplitude. In fact, a closer inspection would reveal that the initial large wave packet contains only upshifted components and the latter only downshifted, as shown in Fig. 5. Once again considering Fig. 4, the delay between ω_p falling below one and the rise in the signal can be explained by the finite propagation time of the waves from the plasma to the observation point.

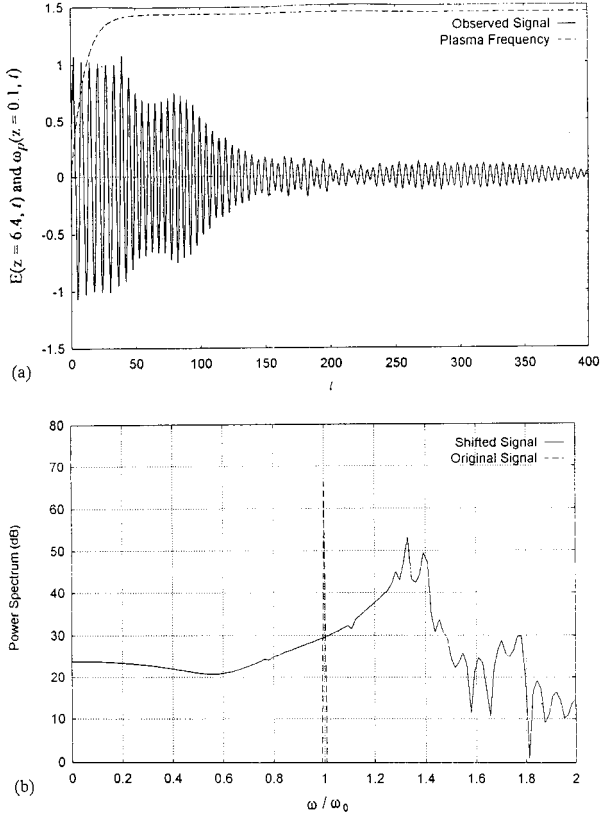


FIG. 6. (a) Time dependence of the observed signal outside the plasma structure with $N=11$, $L=0.6$, $d=0.2$, $\omega_{p0}=1.2$, $\alpha=0.1$, and $\gamma=0$, and (b) its power spectrum.

The power spectrum of this signal reveals the usual upshifted peaks, but also a large downshifted part whose power spectrum varies almost linearly with frequency. This feature has been shown [15] to match quite well with the experimentally observed spectrum. To show that it is the trapping effect that leads to the accumulation of wave energy during the plasma transition period and is responsible for the enhancement of the downshifting process we present in Fig. 6 the case where there is no plasma decay, $\gamma=0$. The plasma transition period is much shorter than the previous case having a plasma decay. We see that the later rise in signal amplitude seen in Fig. 4 does not occur, as the downshifted waves remain trapped. The power spectrum also exhibits only an upshifted part. To show that the downshifted waves do exist and are trapped in the plasma we move the observation point inside the periodic structure, which is again assumed, to never decay. The result is shown in Fig. 7. As one would expect as the downshifted waves are forever trapped, the time amplitude [Fig. 7(a)] after the escape of the upshifted waves remains approximately constant. The power spectrum presented in Fig. 7(b)

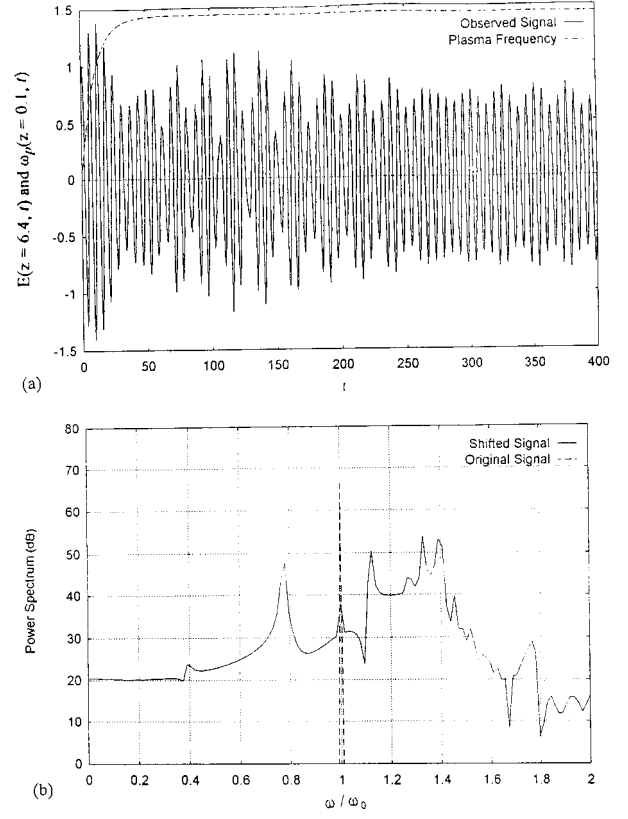


FIG. 7. (a) Time dependence of the observed signal inside the plasma structure with $N=11$, $L=0.6$, $d=0.2$, $\omega_{p0}=1.2$, $\alpha=0.1$, and $\gamma=0$, and (b) its power spectrum.

indeed also contains both upshifted and downshifted lines as that of the previous case presented in Fig. 4(b), except the downshifted spectrum has a sharper distribution and contains much less spectral energy than the counterpart in Fig. 4.

To summarize these points, we have shown that the spatially periodic time varying plasma can trap an electromagnetic wave and alter its spectral content if the plasma frequency rises quickly to a value greater than the wave frequency, and then slowly decays.

III. WAVE IMPEDANCE IN THE SPATIALLY PERIODIC PLASMA

The interpretation of wave trapping phenomenon will be based on the wave impedance Z of the Floquet mode. The wave impedance $Z(z, \beta, \omega_j)$ is defined as $Z=E/H$, where E and H are the phasors of the wave electric and magnetic fields, respectively, of the j th Floquet mode. Since $E(z)$ is given by Eq. (3), $H(z)$ can be derived directly from Faraday's law $(\partial/\partial z)E=i\omega\mu_0H(z)$ as

$$H(z) = \begin{cases} [A \exp(ikz) - B \exp(-ikz)]/Z_0, & -l < z < l \\ \eta \{C \exp[ik\eta(z-l)] - D \exp[-ik\eta(z-l)]\}/Z_0, & l < z < l+d, \end{cases} \quad (10)$$

where $Z_0 = (\mu_0/\epsilon_0)^{1/2}$ is the wave impedance in free space.

With the aid of Eq. (5), the normalized impedance $\bar{Z} = Z/Z_0$ is derived to be

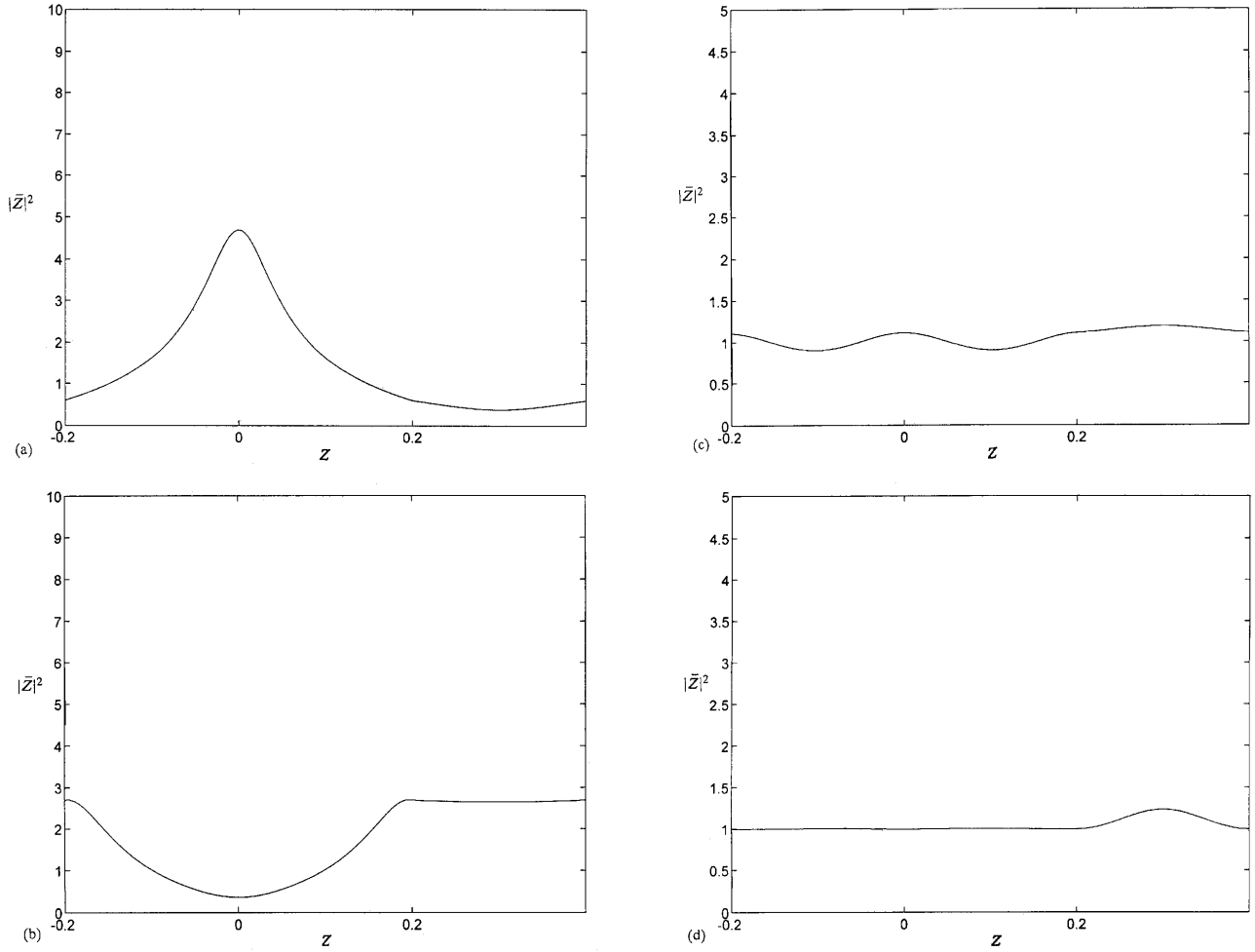


FIG. 8. Amplitude functions of the mode wave impedances in one spatial period of the plasma for the first four Floquet modes corresponding to points 1 to 4 in Fig. 2; (a) frequency downshifted mode $\omega=0.8388$, (b) first frequency upshifted mode $\omega=1.2723$, (c) second frequency upshifted mode $\omega=2.4379$, and (d) third frequency upshifted mode $\omega=2.7583$.

$$\bar{Z} = \begin{cases} \frac{\sqrt{b^2-1} + i \sin 2kz}{b + \cos 2kz}, & -l < z < l \\ \frac{\zeta [\eta - \tan kl \tanh \eta(z-l)] + i \zeta^{-1} [\tan \eta(z-l) + n \tan kl]}{\eta [1 - \eta \tan kl \tanh \eta(z-l)] + i \zeta [\eta \tan \eta(z-l) + \tan kl]}, & l < z < l+d, \end{cases} \quad (11)$$

where $\zeta = \sqrt{b-1}/\sqrt{b+1}$; the $k = \omega/c$ of each Floquet mode is determined by the dispersion relation (6) for a “ β ” value. Thus, $\bar{Z} = \bar{Z}(z; \beta, \omega_j)$ and is a complex function of z , in general. The impedance is, however, a real quantity at the symmetry point $z=0$, and is given by

$$\bar{Z}(0) = \zeta = (1 - \eta^2 \beta_0^2 + \Lambda_0)^{1/2} / (\eta^2 - \beta_0^2 + \Lambda_0)^{1/2},$$

where $\beta_0 = \tan kl$ and $\Lambda_0 = 2 \eta \tan kl \cot k \eta d$.

In Fig. 8, the square $|\bar{Z}|^2$ of the absolute value of the impedances of the first four Floquet modes for the case of $\omega_{p0} = 1.2$ (i.e., corresponding to the case of Fig. 2 and the experimental condition observing wave trapping phenom-

enon) are presented. A characteristic difference among them is observed. For the frequency downshifted mode, the wave impedance is large (>1) in the free space region and small (<1) in the plasma [Fig. 8(a)], and it is distributed in the opposite way for the first frequency upshifted mode [Fig. 8(b)]. The impedances are about 1 for the second and higher frequency upshifted modes [Figs. 8(c) and 8(d)], i.e., the plasma effect on field distributions becomes insignificant because the mode frequencies are much larger than the plasma frequency. The results indicate that the frequency downshifted mode distributes most of its electric energy in the free space while that of the first frequency upshifted mode is distributed in the plasma layers. It is noted that \bar{Z} of the frequency downshifted mode in the periodic structure is a complex function; its phase function is shown in Fig. 9(a).

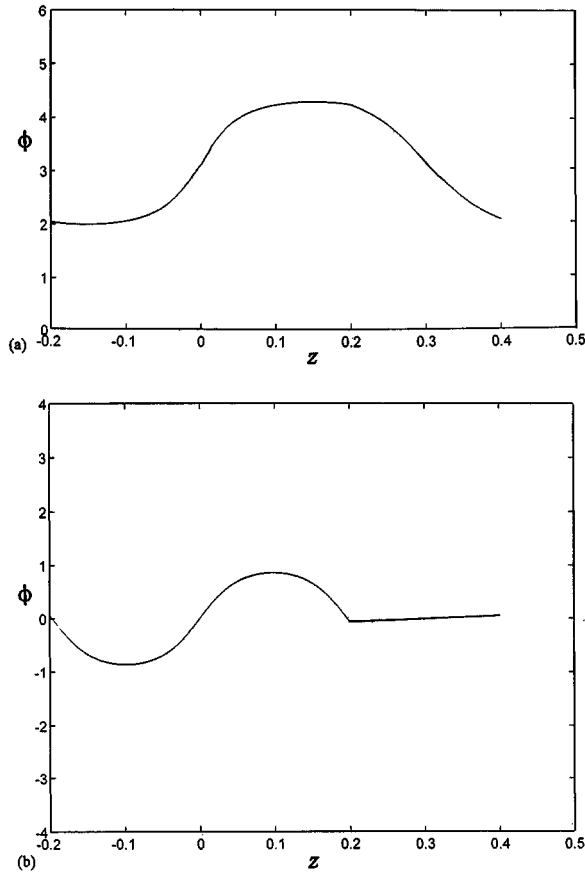


FIG. 9. Phase functions of the mode wave impedances in one spatial period of the plasma, (a) frequency downshifted mode, and (b) first frequency upshifted mode, corresponding to points 1 and 2 in Fig. 2, respectively.

The strong spatial dependence of the phase function suggests that the downshifted mode has a large modulation in its propagation constant. Thus, the frequency downshifted wave is effectively trapped in the periodic structure, in particular, in the free space region between two plasma layers (the propagation of the wave from one region to the other one is via the process of tunneling through the plasma), and can accumulate its mode energy without significant loss caused by the realistic collision damping of the plasma (which is not included in the theory and numerical analysis). On the other hand, \bar{Z} of the frequency upshifted modes are very close to real functions in the plasma regions and complex in the free space regions. The phase of \bar{Z} of the first frequency upshifted mode is shown in Fig. 9(b). Thus, the frequency upshifted modes are quite free to propagate out of the periodic plasma. Moreover, the damping effect of the plasma on them is enhanced, in particular, on the first frequency upshifted mode, by their field distributions. This explains why, in the experiments [15], the frequency downshifted signals were detected repetitively with very convincing results while the frequency upshifted signals were hardly detected or had their spectral intensities very close to the noise level.

IV. SUMMARY

Applying Bloch's theorem, the dispersion relation of an electromagnetic wave propagating in a periodic plasma medium is derived. It consists of branches of dispersion curves, each one represents a Floquet mode and covers a frequency range as a pass band. The frequency gap between two adjacent branches of these dispersion curves forms a band gap. One special feature of the band diagram is that the pass bands extend to the region below the cutoff frequency of a uniform plasma. Therefore, it is shown that a rapidly created periodic plasma can convert a preexisting electromagnetic wave into both frequency upshifted and downshifted new waves. The numerical results further show that the frequency downshifted waves are trapped effectively in the periodic structure. The wave trapping process enables the frequency downshifted waves to accumulate their energy during the finite transition period of the plasma. Consequently, a significant enhancement of their spectral intensities is observed. The vastly improved efficiency in the downshift mechanism due to the trapping effect is important to the application, for example, converting a narrow band continuous (cw) wave into an ultrawide band pulse train by interacting the cw wave with a repetitively created rapidly growing and spatially periodic plasma.

The wave impedance of each Floquet mode in the periodic plasma is derived and used for interpreting the wave trapping phenomenon. The absolute value of the impedance of the frequency downshifted mode is large (> 1) in the free space region and small (< 1) in the plasma layer, but it has an opposite distribution for the frequency upshifted modes. This characteristic difference together with the difference in the spatial variation rates of the phase functions of the impedances explain why only the frequency downshifted waves are trapped and experience reduced collision loss in the plasma layer. On the other hand, the frequency upshifted modes are untrapped and experience enhanced collision loss in the plasma layer. The impedance description provides a physical insight to understand why only the frequency downshifted signals were able to be detected repetitively in the experiments [15] with considerably enhanced spectral intensities while the frequency upshifted signals were missing in the measurements.

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